Disturbance Attenuation and Fault Detection via Zero-pole assignment
-- a dynamic observer approach

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Outline

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• 3. TFMs and Zeros of Dynamic Observers
• 4. Zeros-pole Assignment
• 5. Application and Results
1. Introduction

• Main objective
to present a constructive robust dynamic observer design procedure integrating zero-pole assignment techniques

• Motivation:
Although observer based fault detection theories have become rich, the basic structures are confined in the traditional static observers.

• Limitation: zero invariance of static observers.

• Feature: some zeros of dynamic observer can be assigned arbitrarily.

• Zeros can be used to attenuate the disturbances.
2. Problem Formulation

- Consider a system corrupted by disturbances/faults

\[
\begin{align*}
\dot{x} &= Ax + Bu + B_f f + B_d d \\
y &= Cx + Du + D_f f + D_d d
\end{align*}
\]

- The residual \( r = y - \hat{y} \) works as a fault indication signal and a correct term.
- The observer gain matrix \( K \) is a constant coefficient matrix.
Dynamic observer design via zero-pole assignment

Plant

Actuator fault input

Sensor fault

Sensor noise

Observer

Input noise

$u$, $u^*$, $y^*$, $(A, B, C, D)$

Observer output

$\hat{y}$, $y$, $r$
3. Dynamic Observer

• A m-th order dynamic observer is

\[
\begin{align*}
\dot{z}_{k+1} &= K_1 z_k + K_2 (y - C \hat{x}_k) \\
 r_k &= K_3 z_k + K_4 (y - C \hat{x}_k)
\end{align*}
\]

\[
\begin{align*}
\dot{x}_{k+1} &= A \hat{x}_k + B u_k + s_k \\
\hat{y}_k &= C \hat{x}_k
\end{align*}
\]

• Difference: the feedback path
constant numerical matrix V.S. dynamic system

• Transfer Function Matrix (TFM):

\[
r \rightarrow v : \quad H(s) = K_3 (sI - K_1)^{-1} K_2 + K_4
\]
TFMs of Dynamic Observer

- Define $e = x - \hat{x}$,

\[
\begin{pmatrix}
\tilde{e}(t) \\
\tilde{x}(t)
\end{pmatrix} = 
\begin{bmatrix}
A - K_4 \tilde{C} & K_3 \\
K_2 \tilde{C} & K_1
\end{bmatrix} 
\begin{pmatrix}
\hat{e}(t) \\
\hat{x}(t)
\end{pmatrix} 
+ 
\begin{bmatrix}
B_d & K_4 D_d \\
\tilde{B}_d & \tilde{K}_4 \tilde{D}_d
\end{bmatrix} d(t) + 
\begin{bmatrix}
B_f & K_4 D_f \\
\tilde{B}_f & \tilde{K}_4 \tilde{D}_f
\end{bmatrix} f(t)
\]

\[
r(t) = [C \tilde{O}] \begin{pmatrix}
e(t) \\
z(t)
\end{pmatrix} + \tilde{D}_d d(t) + \tilde{D}_f f(t)
\]

s-transform

\[
r(s) = G_f(s)f(s) + G_d(s)d(s)
\]

\[
\begin{align*}
G_f(s) &= \tilde{C}(sI - \tilde{A})^{-1} \tilde{B}_f + \tilde{D}_f \\
G_d(s) &= \tilde{C}(sI - \tilde{A})^{-1} \tilde{B}_d + \tilde{D}_d
\end{align*}
\]

(12)
Poles and zeros

Poles: eigenvalue \( (\tilde{A}) \)

Zeros in multivariable system:

TFM loses rank locally

Z1: (disturbance zeros of the actual system)

\[ Z_1 = \{ s \mid \text{rank } G_d (s) < \min( r, d) \} \]

Z2: (disturbance zeros of the dynamic observer)

\[ Z_1 = \{ s \mid \text{rank } \tilde{G}_d (s) < \min( r, d) \} \]
Theorem 1

- $Z_2$ are the disturbance zeros $Z_1$ together with the eigenvalues of $K_1$

$$Z_2 = Z_1 \cup \{ \text{eigenvalues of } K_1 \}$$

- **R1:** zeros are invariant in static observer

- **R2:** a $m$th-order dynamic observer introduces $m$ additional zeros

- **R3:** Those additional zeros are assignable arbitrarily by assigning the poles of the feedback.
4. Zero-pole assignment
1. Estimate the disturbance frequency \( \{\omega_r\} \) via residual spectrum analysis.
2. Zero-assignment: determine the order and assign the eigenvalues of \( K_1 \) close to \( \{\omega_r\} \)
3. Specify the pole’s region (far from \( \{\omega_r\} \))
4. Optimize the performance index
\[
\rho + \| G_d(s) \|_{s=0} - \| G_f(s) \|_{s=j\omega_r} = 0
\]
5. Simulation results

- Consider a 2-in-2-output system

\[
\begin{cases}
\dot{x}(t) = \begin{bmatrix}
-0.943 & 0.1601 \\
3.9439 & -3.234 \\
\end{bmatrix} x(t) + \begin{bmatrix}
86.794 & 40.312 \\
154.691 & 81.275 \\
\end{bmatrix} u(t) \\
y = \begin{bmatrix}
1 \\
0 \\
0 \\
1 \\
\end{bmatrix} x(t)
\end{cases}
\]

- Disturbance and fault model
(input disturbances and actuator faults)

\[
B_d = B \\
D_d = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
\end{bmatrix}
\]

\[
B_f = B \\
D_f = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
\end{bmatrix}
\]
Dynamic observer design via zero-pole assignment

Disturbances & frequency estimation

- disturbances

\[
d(t) = \begin{pmatrix} d_1(t) \\ d_2(t) \end{pmatrix} = \begin{pmatrix} 0.5\sin(5t) \\ 0.4\cos(5t) \end{pmatrix}
\]
• $\omega_r = 5$ rad/sec, assign the zeros to $\pm 5j$
• Two zeros required, then $m=2$

\[
K_1 = \begin{bmatrix}
0 & -5 \\
+5 & 0
\end{bmatrix}
\]

• Set the desired regions of poles:
  (a) their real parts $< -1$
  (b) their imaginary parts close to 0

• Set the initial value randomly and optimize the performance index

\[
K_2 = \begin{bmatrix}
0.1430 & -2.6552 \\
-4.3399 & 2.8362
\end{bmatrix}, \quad K_3 = \begin{bmatrix}
-1.1410 & 1.0494 \\
-5.4621 & -0.590
\end{bmatrix}, \quad K_4 = 0_{2\times2}
\]
Dynamic observer design via zero-pole assignment

TFM $G_d(s)$ and $G_f(s)$

- Because $B_d = B_f = B$, $D_d = D_f = 0$, then $G_d(s) = G_f(s)$

Dynamic Filter ($K_4 = 0$)

From: $d_1$ or $f_1$

From: $d_2$ or $f_2$
Dynamic observer design via zero-pole assignment

Residuals without fault

$\|r(t)\|_{\text{dyn}}$

$\|r(t)\|_{\text{place}}$

Time (second)

magnitude
$f_1(t) = \begin{cases} 0 & (t < 20) \\ 0.05 & (t \geq 20) \end{cases} \quad f_2(t) = \begin{cases} 0 & (t < 30) \\ 0.05 & (t \geq 30) \end{cases}$
Residuals of incipient faults

Plant outputs under the incipient actuator fault

Residuals of the incipient actuator fault

\[ y_2(t) \]

\[ y_1(t) \]
Thanks